

# SAMPLE PAPER 6

## Leaving Certificate

# Mathematics

## Paper 1

## Ordinary Level

**Time:** 2 hours, 30 minutes

300 marks

Examination number
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Centre stamp
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Running total	
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For examiner	
Question	Mark
1	
2	
3	
4	
5	
6	
7	
8	
9	
Total	

Grade
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## Instructions

There are **two** sections in this examination paper.

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer all nine questions.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the *Formulae and Tables* booklet. You must return it at the end of the examination. You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

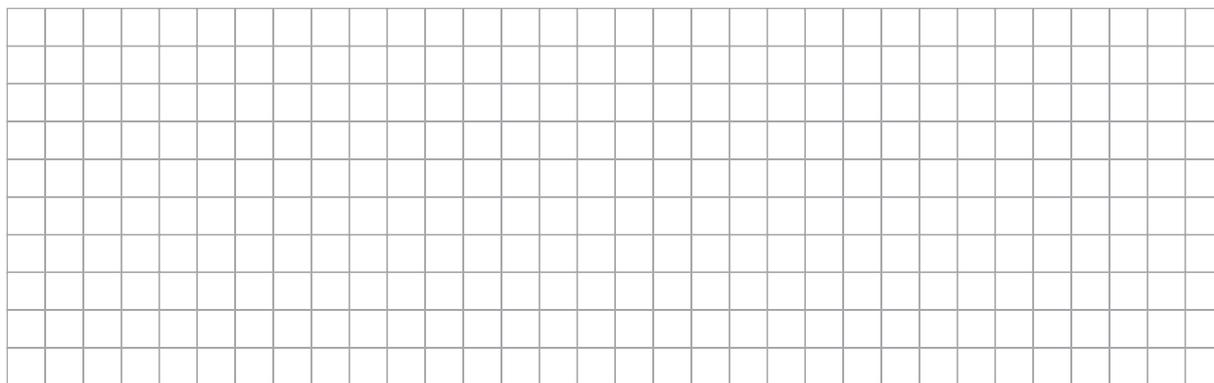
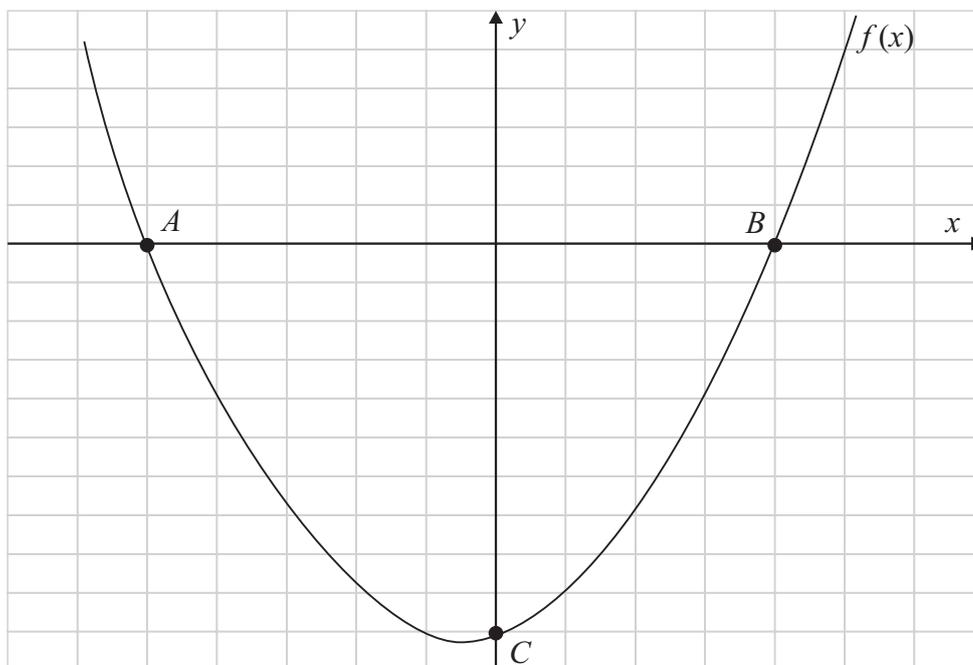
Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

**Question 1**

**(25 marks)**

- (a) The graph of  $f(x) = x^2 + x - 20$  crosses the  $x$ -axis at  $A$  and  $B$ . Find the coordinates of  $A$  and  $B$ . Find the coordinates of  $C$ , the point where the graph cuts the  $y$ -axis. Draw up a suitable scale for the  $x$  and  $y$  axes below by numbering the grid appropriately.



- (b) The graph of the line  $g(x)$  has  $g(0) = 0$  and  $g(1) = 2$ . Plot  $g(x)$  on the grid above. Use this graph to find the two values of  $x$  for which  $f(x) = g(x)$ .



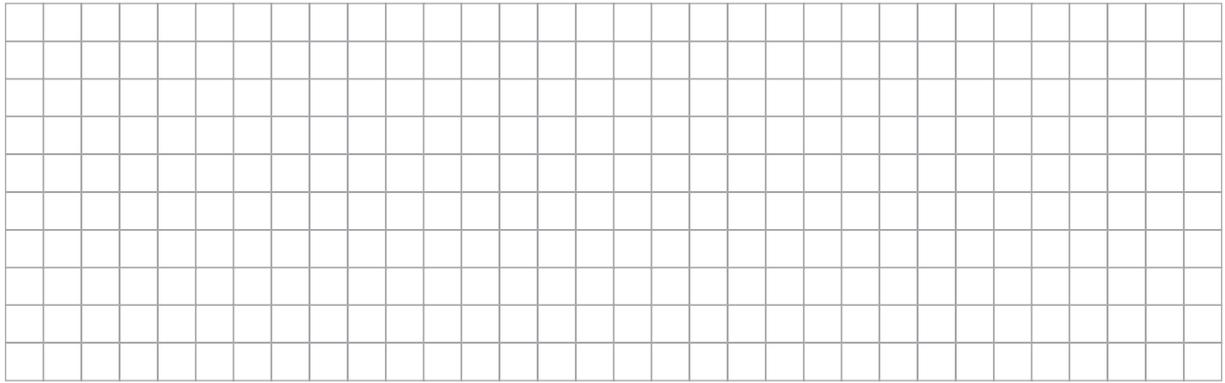
- (c) Find the equation of  $g(x)$  in the form  $ax + b$ .

**Question 2**

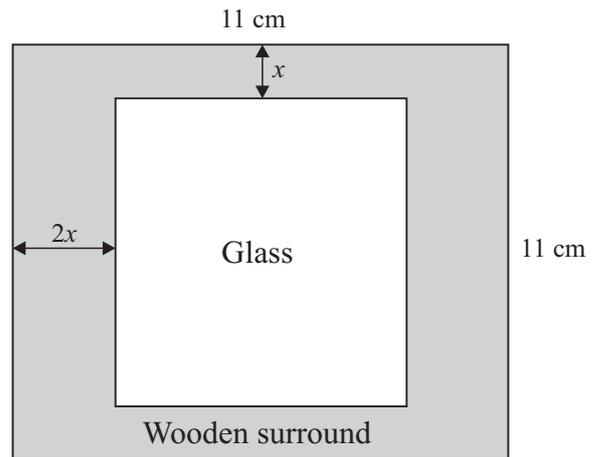
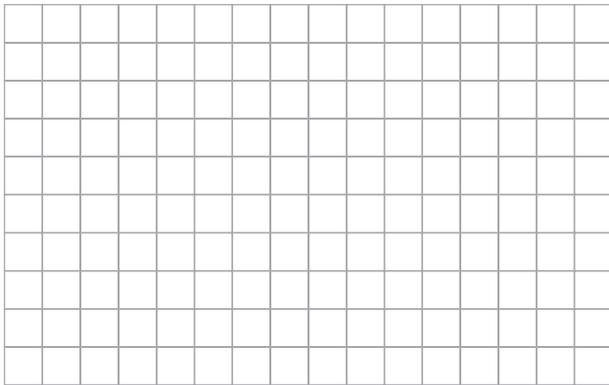
**(25 marks)**

- (a) If  $f(x) = 3\sqrt{x}$ , find  $f(12)$  in its simplest surd form.

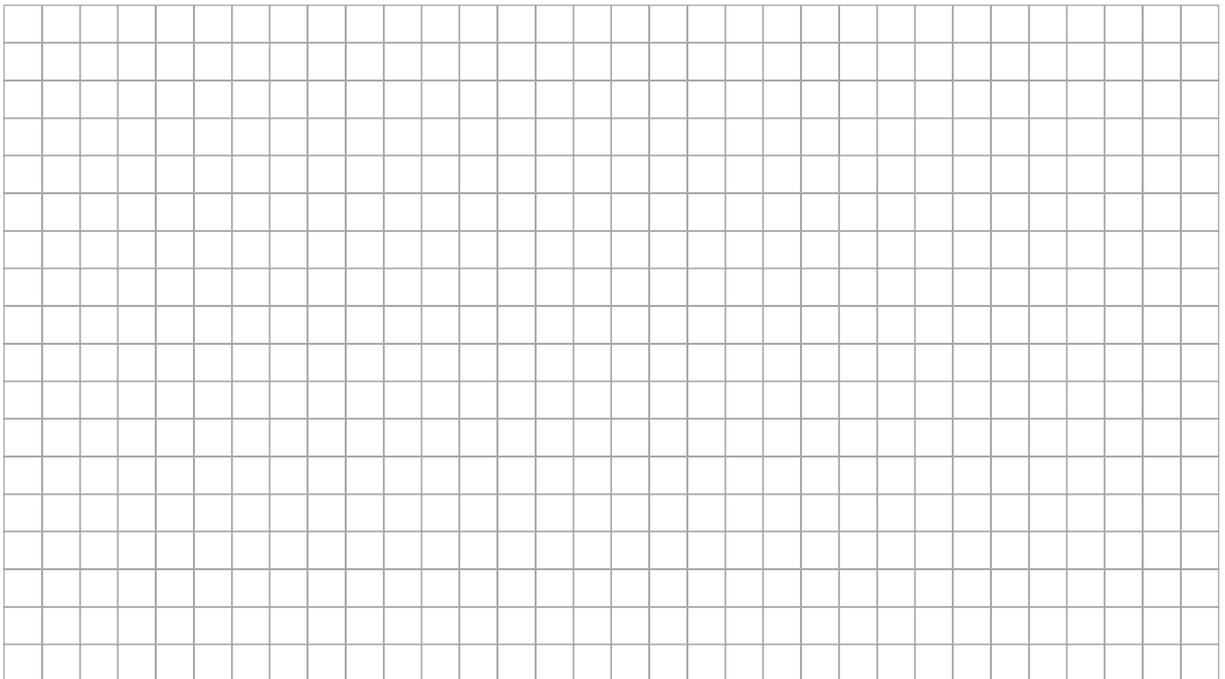
- (b) 3 apples and 4 oranges cost €1.30. 4 apples and 2 oranges cost €1.20. Find the cost of an apple and an orange. Find the cost of 6 apples and 6 oranges.



- (c) (i) The frame of a photograph is 11 cm by 11 cm. The wooden surround has a width of  $2x$  cm on the sides and  $x$  cm on the top and the bottom. Show that the area of the glass is given by  $121 - 66x + 8x^2$ .



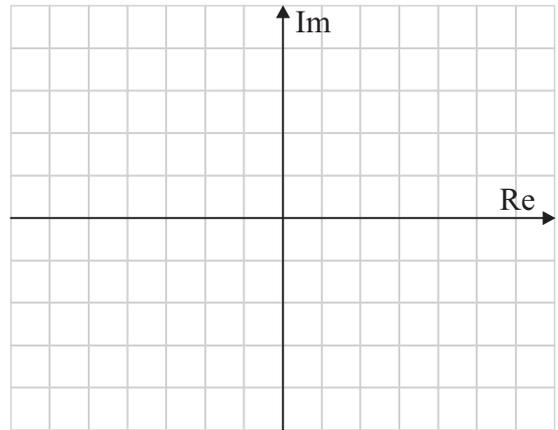
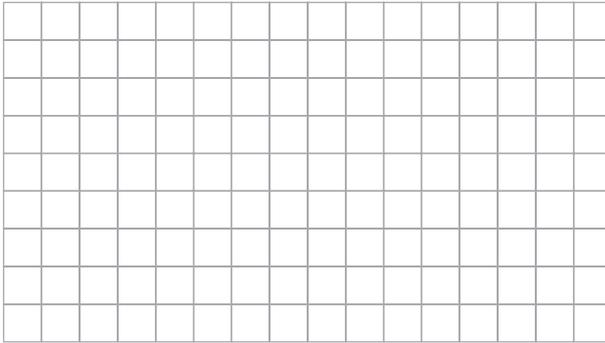
- (ii) If this area is  $40 \text{ cm}^2$ , find  $x$ .



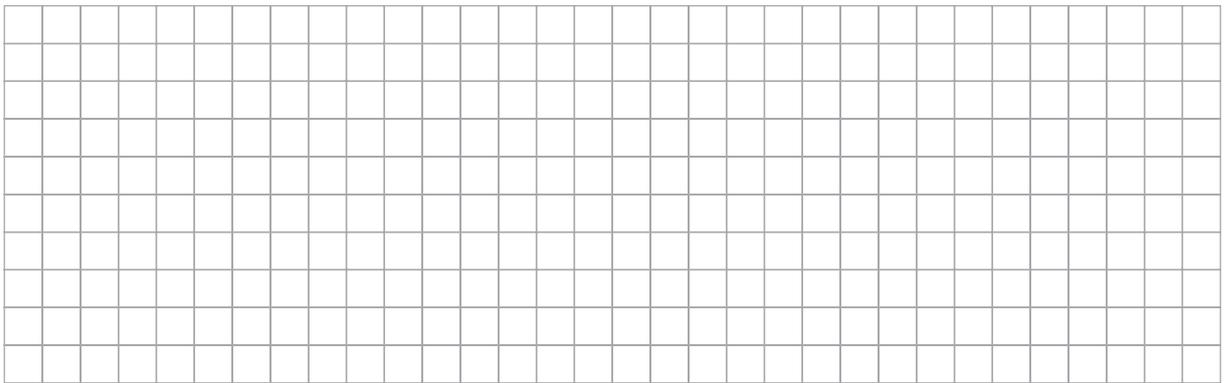
**Question 3**

**(25 marks)**

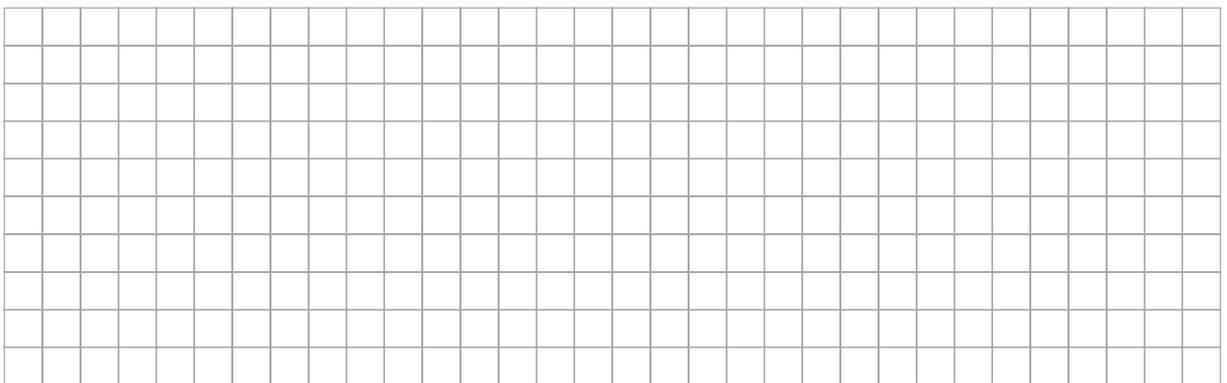
- (a) (i) If  $z = 6 - 4i$ , where  $i = \sqrt{-1}$ , plot  $z$  and  $-z$  on the Argand diagram.



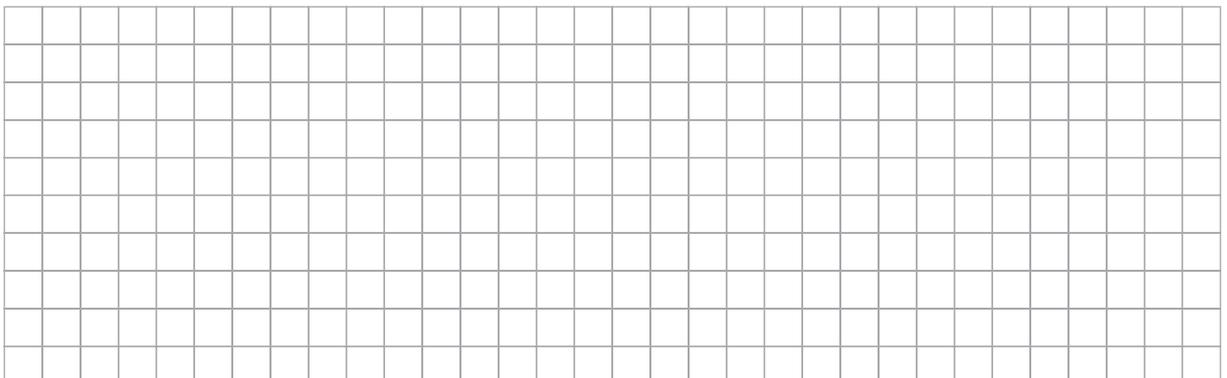
- (ii) Find  $|z|$  and  $|-z|$ . Why is  $|z| = |-z|$ ?



- (b) (i) If  $z = 3 - 2i$ , find  $\bar{z}$ . Evaluate  $z\bar{z}$  and  $\frac{z}{\bar{z}}$ .



- (ii) Find  $\left| \frac{z}{\bar{z}} \right|$ .



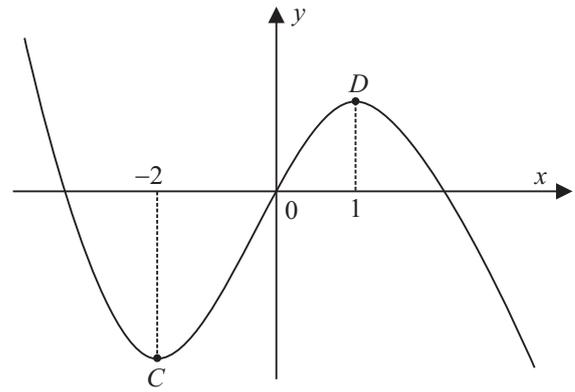
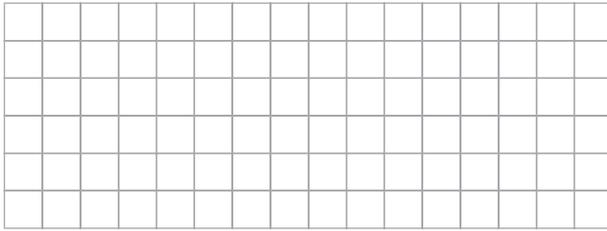


**Question 5**

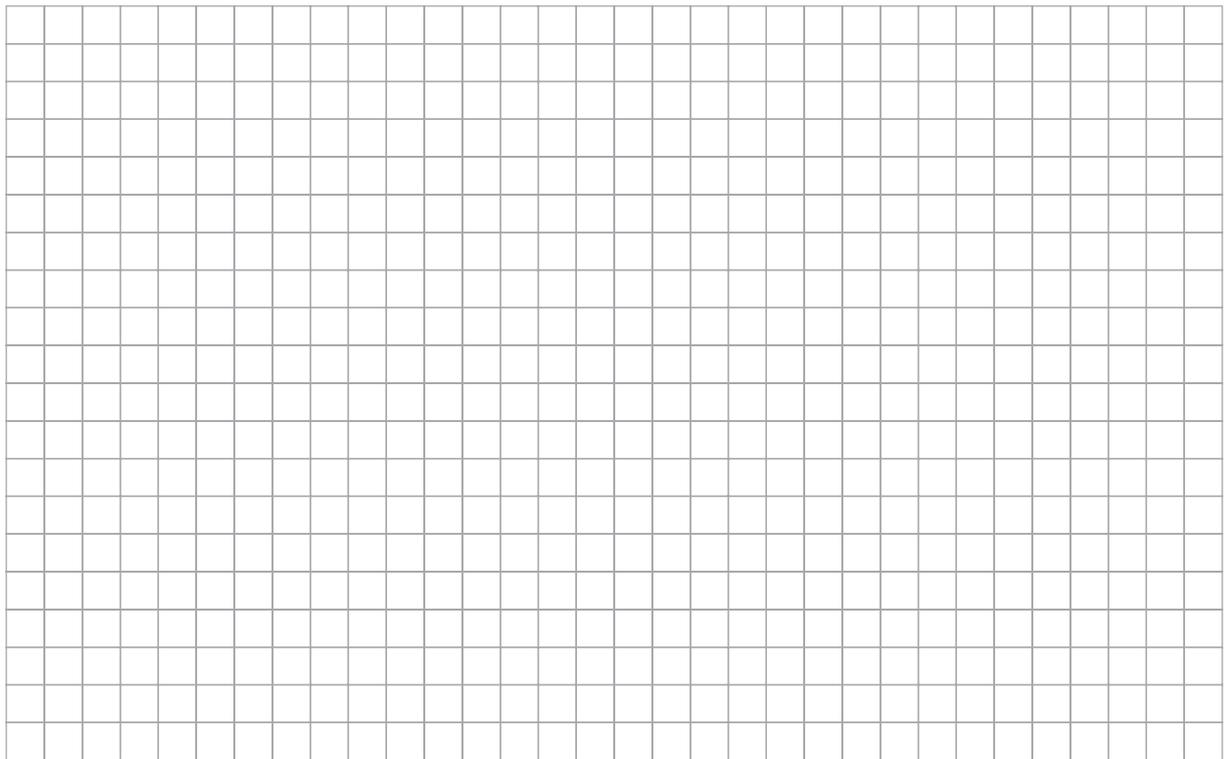
**(25 marks)**

The curve  $y = -x^3 + ax^2 + bx$  shown has a local minimum at  $x = -2$  and a local maximum at  $x = 1$ .

(a) Show the curve passes through  $(0, 0)$ .



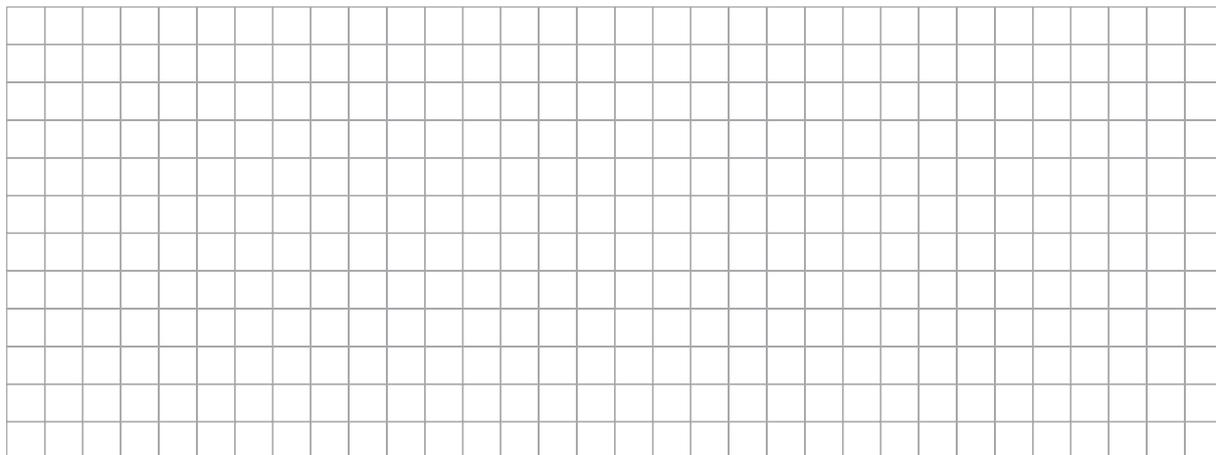
(b) Show that  $a = -\frac{3}{2}$  and  $b = 6$ .



(c) Find the coordinates of the points  $C$ , the local minimum, and  $D$ , the local maximum.



(d) Find the equation of the tangent at (0, 0).



**Question 6**

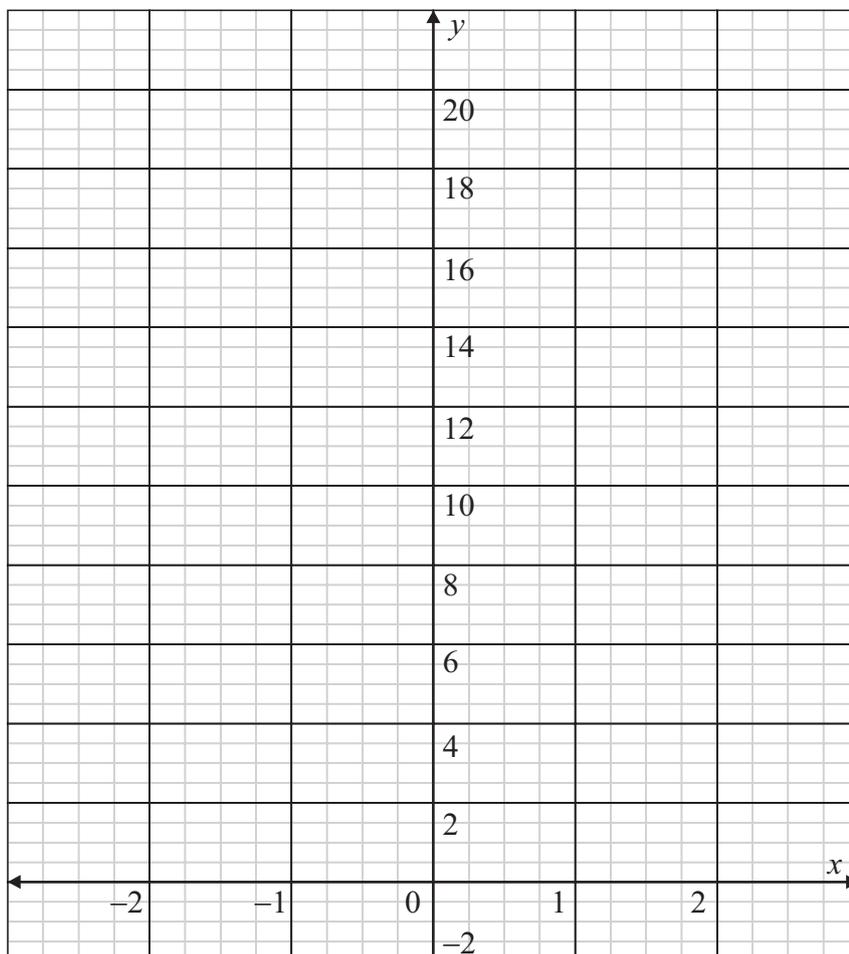
**(25 marks)**

Let  $f(x) = 4x^2 + 2x$  and  $g(x) = 3 \times 2^x$ .

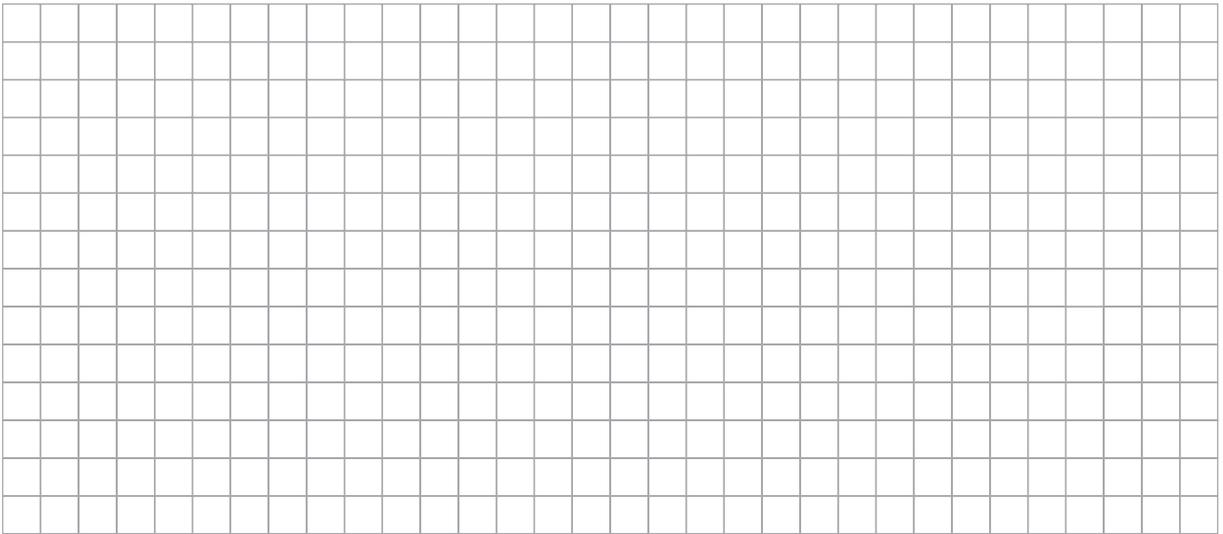
(a) Complete the table below.

$x$	-2	-1	0	1	2
$f(x)$					
$g(x)$					

(b) Plot both graphs on the grid below.



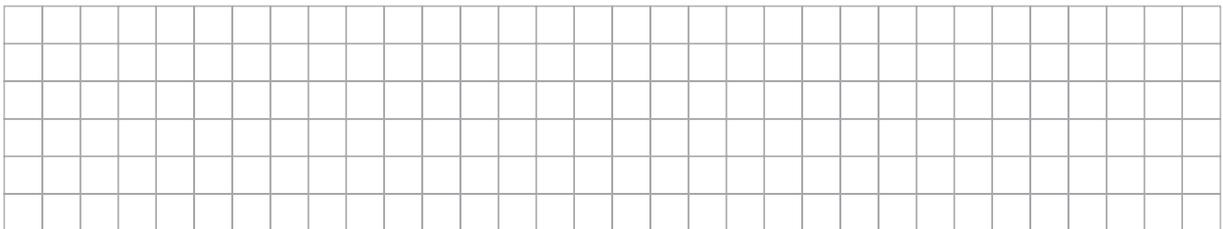
- (c) Find the coordinates of the local minimum of  $f(x)$  using calculus. Check your answer using the graph.



ALGEBRA: Local minimum \_\_\_\_\_

GRAPH: Local minimum \_\_\_\_\_

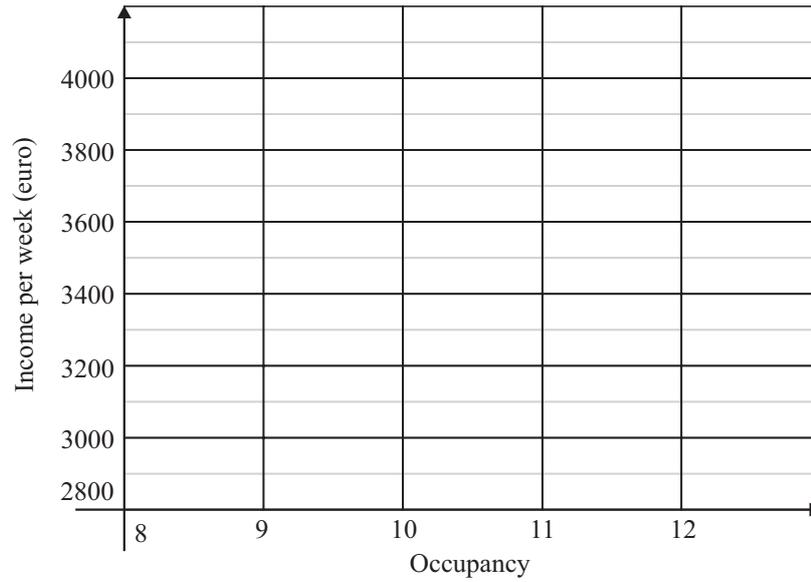
- (d) Use your graphs to estimate the solutions of  $f(x) = g(x)$ .



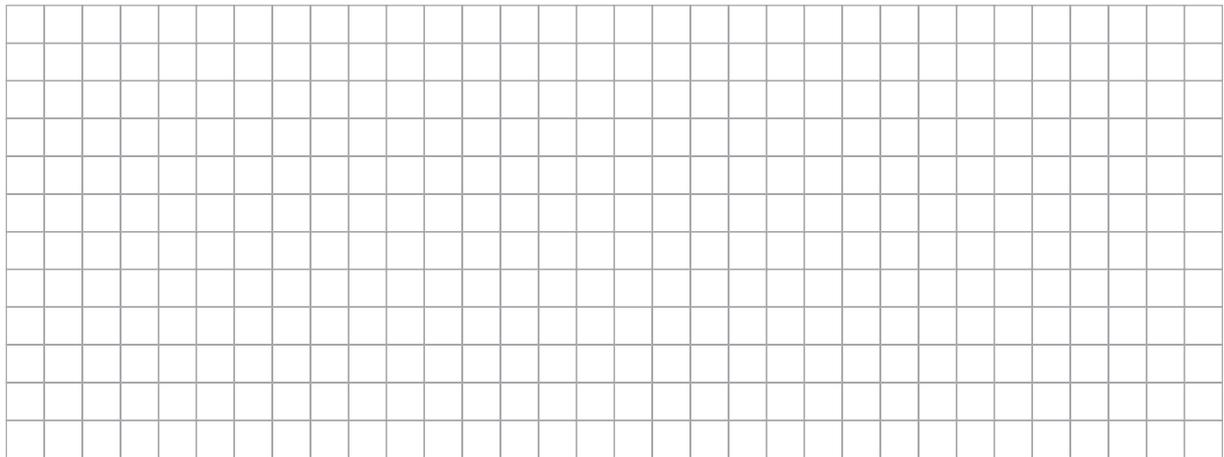




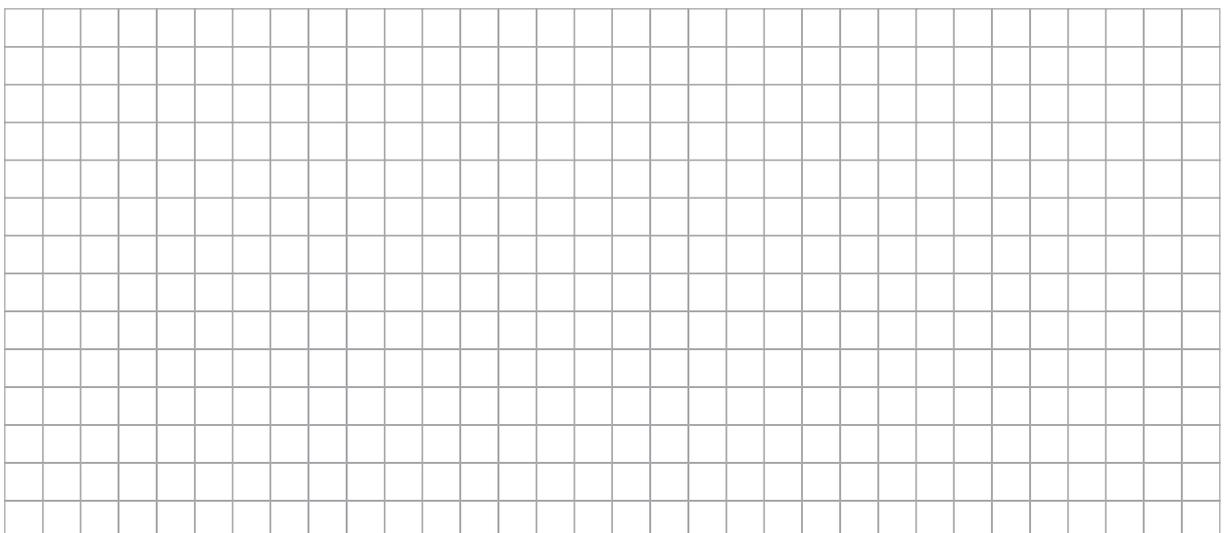
(ii) Plot a graph on the grid below of income,  $I$ , versus occupancy per week,  $x$ , for  $8 \leq x \leq 12$ .



(iii) Use the graph to find an equation connecting  $I$  and  $x$ .



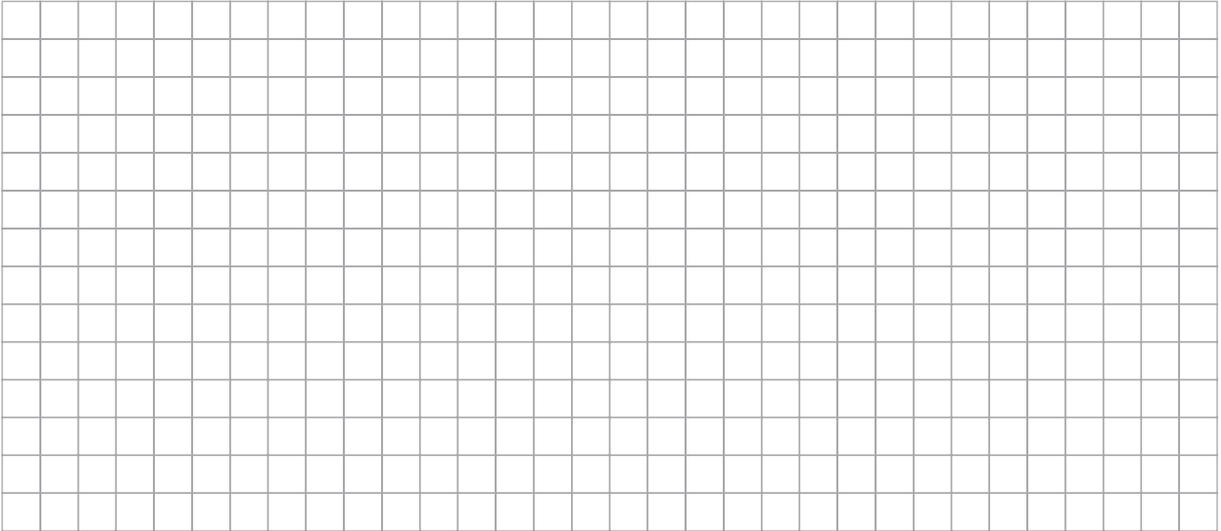
(iv) If the Bravo owner increased his house to 5 bedrooms with maximum occupancy of 16, based on 10 sharing, what is the new equation?



**Question 8****(50 marks)**

The first four odd numbers are 1, 3, 5, 7, ....

- (a) Find the general term  $T_n$  for those odd numbers.



- (b) The odd numbers can all be written as the difference of two squares:

$$1 = 1^2 - 0^2$$

$$3 = 2^2 - 1^2$$

$$5 = 3^2 - 2^2$$

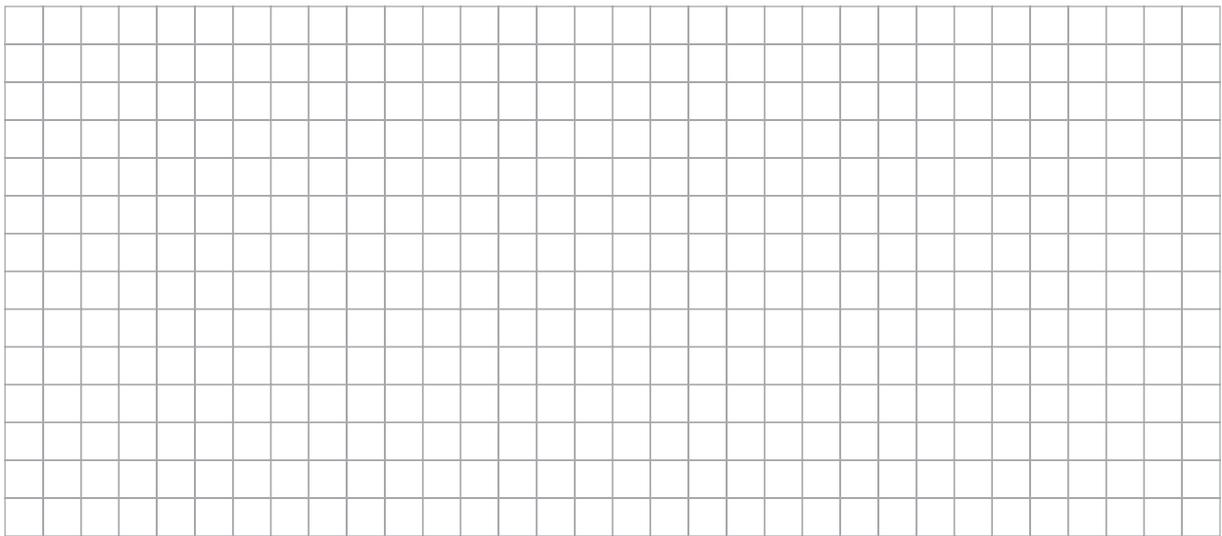
Express 7 in this form.

$$7 = \underline{\hspace{4cm}}$$

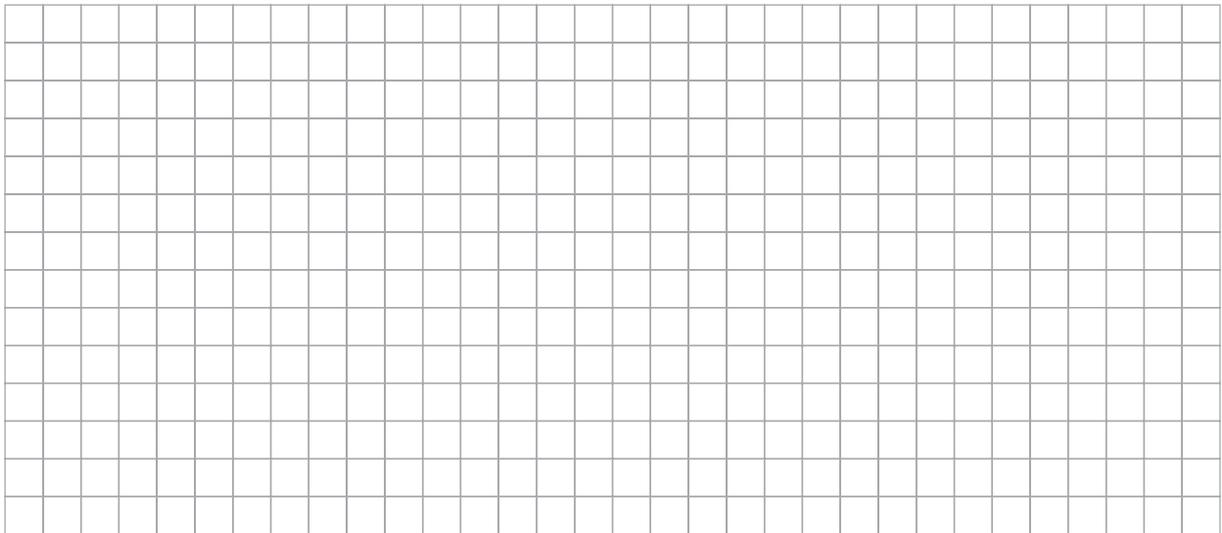
- (c) What term in the sequence is 21? Express 21 as the difference of two squares.



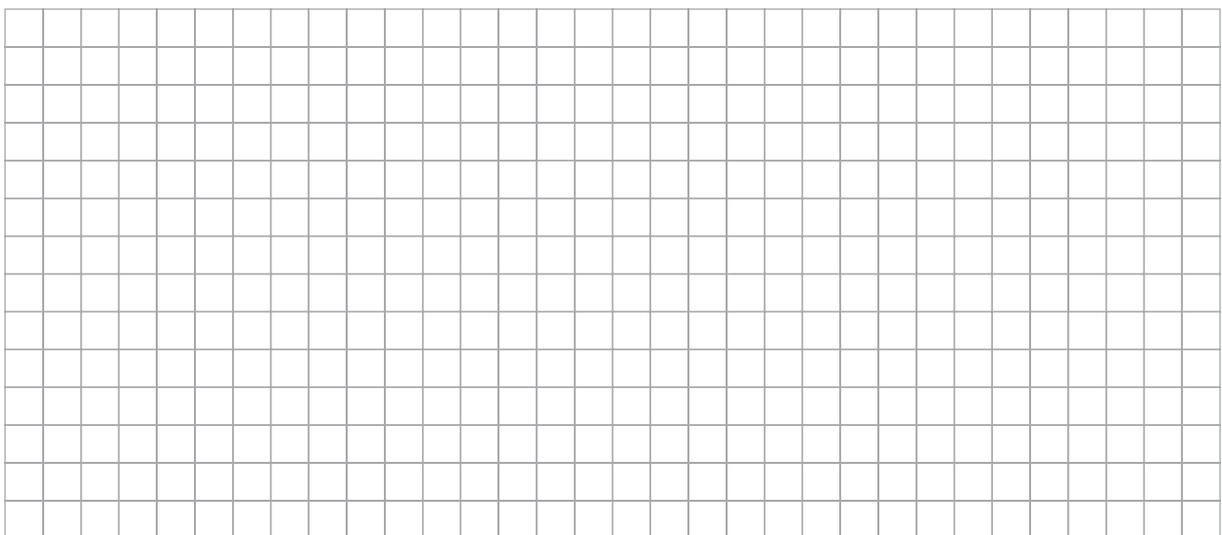
- (d) Express the  $n^{\text{th}}$  odd number as the difference of two squares and show it is equal to  $T_n$  in part (a).



- (e) Show that the product of two consecutive odd numbers is always odd.



- (f) Find the sum of the first 100 odd numbers.

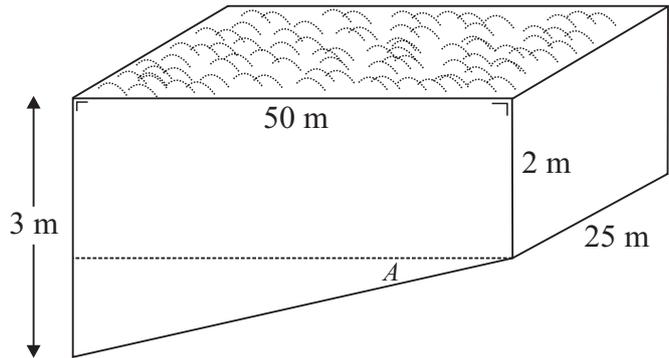
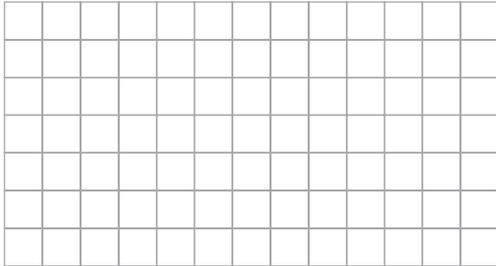


**Question 9**

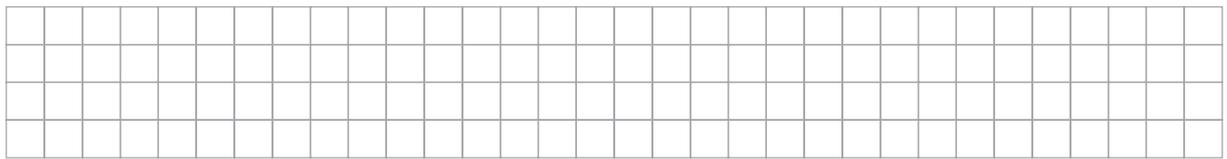
**(50 marks)**

A diagram of an Olympic swimming pool is shown (not to scale).

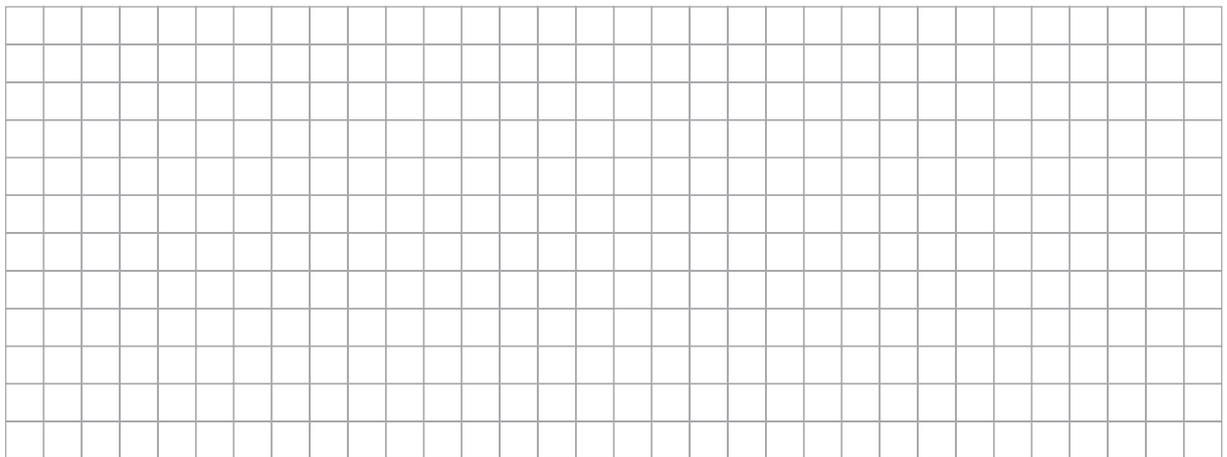
- (a) Find the angle  $A$  the sloping floor of the pool makes with the horizontal, to three decimal places.



- (b) Find the surface area of the water in the pool.

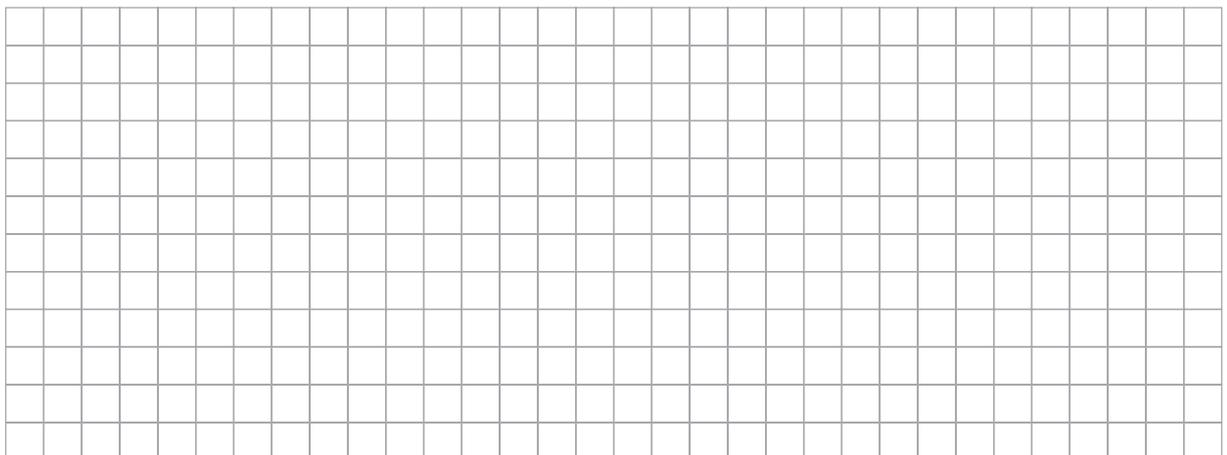


- (c) Find the volume of water required to fill the pool.

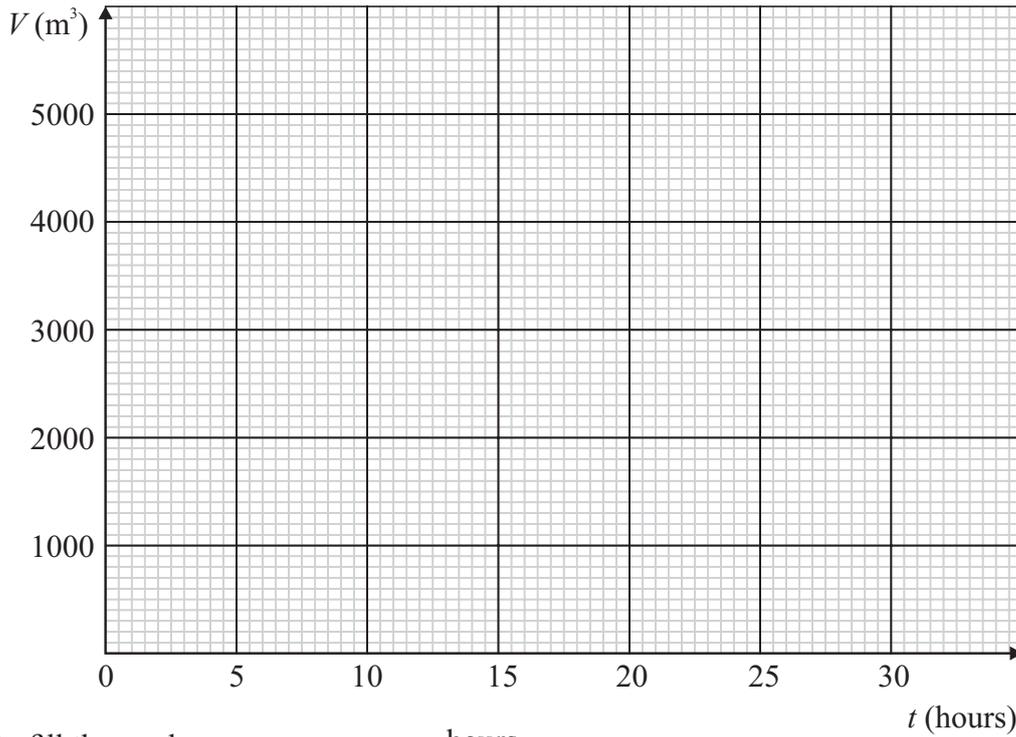


- (d) The pool is filled by a pump according to the equation  $V = 80 + 5t^2$ , where  $V$  is the volume in metres cubes ( $\text{m}^3$ ) after time  $t$  in hours (h). Complete the table:

$t$ (h)	0	5	10	15	20	25	30
$V$ ( $\text{m}^3$ )							

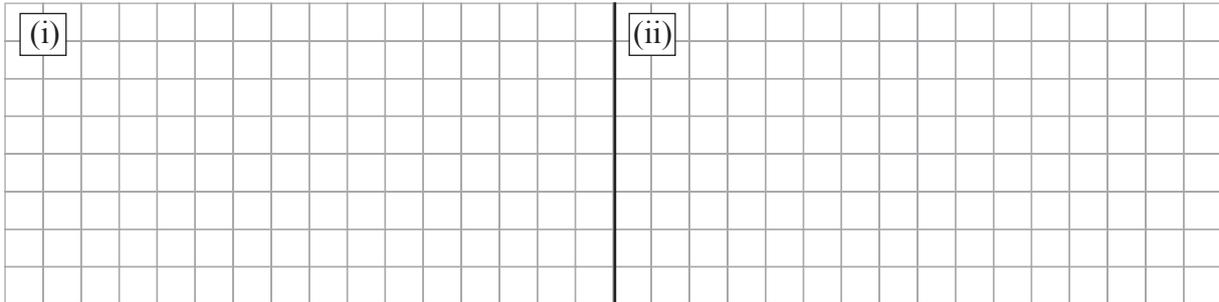


(e) Plot a graph of  $V$  against  $t$ . Use the graph to estimate the time to fill the pool to the nearest hour.



Time to fill the pool = \_\_\_\_\_ hours

(f) Find the rate at which the pool is filled at (i)  $t = 6$  hours, (ii)  $t = 16$  hours.



(g) If the pump was turned off after 12 hours due to overheating, what percentage of the pool was filled after 12 hours? If it was then filled at a steady rate of  $85 \text{ m}^3$  per hour not using the pump, how long more would it take to fill the pool to the nearest minute?

